

CS_375/CS_M75 Logic for Computer Science

Coursework 1

Due date: Thursday, 24th of October 2013

Question 1. Let $\text{atom}(F)$ be the number of occurrences of atomic formulas in F and let $\text{con}(F)$ be the number of occurrences of logical connectives ($\neg, \wedge, \vee, \rightarrow$) in F .

Give definitions of $\text{atom}(F)$ and $\text{con}(F)$ by recursion on formulas. [20 marks]

Question 2. Prove $\text{atom}(F) \leq 2 \cdot \text{con}(F) + 1$. [20 marks]

Question 3. Which of the following statements are true?

- (a) If $F \rightarrow G$ is a tautology and F is a tautology, then G is a tautology.
- (b) If $F \rightarrow G$ is a tautology and F is satisfiable, then G is satisfiable.
- (c) If $F \rightarrow G$ is satisfiable and F is a tautology, then G is satisfiable.
- (d) If $F \rightarrow G$ is satisfiable and F is satisfiable, then G is satisfiable.

Justify you answers. [20 marks]

Question 4. (Uwe Schoening “Logic for Computer Science”, Exercise 11). Consider the following statements:

- (a) Everybody having a musical ear is able to sing properly.
- (b) Nobody is a real musician if they cannot electrify their audience.
- (c) Nobody who does not have a musical ear can electrify their audience.
- (d) Nobody, except a real musician, can compose a symphony.

Question: Which properties does a person have who has composed a symphony?

To find the answer first formalise the statements.

[20 marks]

Please turn over

Question 5. Consider the logic gate $g : \{0, 1\}^3 \rightarrow \{0, 1\}$ defined by

$$g(x, y, z) := (x + y - 1)^2 \cdot z$$

Find formulas in CNF and DNF defining g .

Also, try to find a formula defining g that has only 5 occurrences of logical connectives.

[20 marks]

Question 6*. (Craig's Interpolation Theorem) Prove that if $F \rightarrow G$ is a tautology, then there exists a formula H containing only atomic formulas occurring in both F and G such that $F \rightarrow H$ and $H \rightarrow G$ are tautologies.

Hint: Use induction on the number of atomic formulas occurring in F but not in G .

[40 marks]

100 marks count as full marks. Question 6* is more difficult than the other questions.